



1)  $\vec{a}, \vec{b}$  **Collinear:**  $\vec{a} = K\vec{b}$        $(a_x, a_y) = K(b_x, b_y)$        $(a_x, a_y, a_z) = K(b_x, b_y, b_z)$

2)  $Comp_{\vec{b}}\vec{a} = |\vec{a}| \cos(\theta)$        $Comp_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$        $Proj_{\vec{b}}\vec{a} = |\vec{a}| \cos(\theta) \frac{\vec{b}}{|\vec{b}|}$        $Proj_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$

3) **Dot ( Scalar ) Product**       $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\theta)$        $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$        $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

1)  $\vec{a} \cdot \vec{a} = |\vec{a}|^2$       2)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$       3)  $\vec{a} \cdot \vec{b} = 0$  **IFF**  $\vec{a} \perp \vec{b}$       4)  $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$       5)  $K\vec{a} \cdot M\vec{b} = KM(\vec{a} \cdot \vec{b})$

4) **Cross ( Vector ) Product**       $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin(\theta)$        $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$

1)  $\vec{a} \times \vec{a} = \vec{0}$       2)  $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$       3)  $\vec{a} \times \vec{b} = \vec{0}$  **IFF**  $\vec{a} = K\vec{b}$       4)  $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$       5)  $K\vec{a} \times M\vec{b} = KM(\vec{a} \times \vec{b})$

**note:**  $\vec{a} \times \vec{b}$  is a new vector that is perpendicular to the plane containing vectors  $\vec{a}$  and  $\vec{b}$  and forms a RIGHT hand system.

5) **Angle:**  $\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$        $\cos(\theta) = \frac{a_x b_x + a_y b_y}{\sqrt{a_x^2 + a_y^2} \sqrt{b_x^2 + b_y^2}}$        $\cos(\theta) = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \sqrt{b_x^2 + b_y^2 + b_z^2}}$   
 $\sin(\theta) = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|}$        $\sin(\theta) = \frac{\sqrt{(a_y b_z - a_z b_y)^2 + (a_z b_x - a_x b_z)^2 + (a_x b_y - a_y b_x)^2}}{\sqrt{a_x^2 + a_y^2 + a_z^2} \sqrt{b_x^2 + b_y^2 + b_z^2}}$

6) **Area of Parallelogram** formed by  $\vec{a}, \vec{b}$ :  $Area = |\vec{a} \times \vec{b}|$        $Area = \sqrt{(a_y b_z - a_z b_y)^2 + (a_z b_x - a_x b_z)^2 + (a_x b_y - a_y b_x)^2}$

7) **Triple Scalar Product:**  $\vec{a} \times \vec{b} \cdot \vec{c} = (a_y b_z - a_z b_y) c_x + (a_z b_x - a_x b_z) c_y + (a_x b_y - a_y b_x) c_z$

8) **Volume of Parallelepiped:**  $Vol = |\vec{a} \times \vec{b} \cdot \vec{c}|$        $Vol = |(a_y b_z - a_z b_y) c_x + (a_z b_x - a_x b_z) c_y + (a_x b_y - a_y b_x) c_z|$   
 formed by  $\vec{a}, \vec{b}$  and  $\vec{c}$

**NOTE:** if volume is **0** then the three vectors are **COPLANAR**

9) **Resolve Vector Componets:**

- a) along x axis:  $|\vec{a}| \cos(\theta)$
- b) along y axis:  $|\vec{a}| \sin(\theta)$

