To Sketch the Curve $y = f(x)$

A) *Domain*

- Determine the set of values of x for which f(x) is defined.

B) *Intercepts*

- Find the y intercept by setting x = 0 and solving for y (0 , y)
- Find the x intercept by setting y = 0 and solving for x (x , 0) *** Do not attempt if too difficult.**

C) *Symmetry*

- If f(-x) = f(x) then f(x) is even and symmetrical about the y - axis.
- If f(-x) = -f(x) then f(x) is odd and symmetrical about the origin.

D) *Asymptotes*

- **Vertical** Equate the denominator of f(x) to zero after dividing out common factors to find $x = x_v$. Evaluate $\lim_{x \to x_v^-} f(x)$ and $\lim_{x \to x_v^+} f(x)$ to identify either $-\infty$ or ∞.

- **Horizontal** If $\lim_{x \to -\infty} f(x) = \mathbf{L}$ or $\lim_{x \to +\infty} f(x) = \mathbf{L}$ then $y = \mathbf{L}$ is a horizontal asymptote.

- **Slant** The line y = **mx + b** is a slant asymptote when $\lim_{x \to \pm\infty} [\, f(x) - (mx + b)\,] = 0$.

E) *Intervals*

- Find all critical numbers $\mathbf{x_c}$ by setting f '(x) = 0 and solving for x and finding all values of $\mathbf{x_c}$ where f '(x) does not exist. Set up a chart of intervals using these critical numbers.
- When f '(x) > 0 the interval is increasing and when f '(x) < 0 the interval is decreasing.

F) *Local Extrema*

- For all values of $\mathbf{x_c}$ obtained by setting f '(x) = 0 and solving for x, use the *Second Derivative Test*. If f "($\mathbf{x_c}$) > 0 then $\mathbf{x_c}$ is a local min and if f "($\mathbf{x_c}$) < 0 then $\mathbf{x_c}$ is a local max. If f "($\mathbf{x_c}$) = 0 or f "($\mathbf{x_c}$) doesn't exist, use the *First Derivative Test*. Remember to then substitute the $\mathbf{x_c}$ back into f(x) to obtain the y values.

- For all values of $\mathbf{x_c}$ obtained by looking for values of x where f '(x) doesn't exist, use the *First Derivative Test*. If the interval changes from inc to dec then $\mathbf{x_c}$ is a local max and if the interval changes from dec to inc then $\mathbf{x_c}$ is a local min. Remember to substitute the $\mathbf{x_c}$ back into f(x) to obtain the y values. These type of local max/min will be a *Cusp*.

G) *Concavity*

- Find all inflect numbers $\mathbf{x_I}$ by setting f "(x) = 0 and solving for x and finding all values of $\mathbf{x_I}$ where f "(x) does not exist. Set up a chart of intervals using these inflect numbers.
- If f "(x) > 0 the interval is concave up and if f "(x) < 0 the interval is concave down.

H) *Inflection*

- Find all values of $\mathbf{x_I}$ by setting f "(x) = 0 and solving for x and evaluating all cases of $\mathbf{x_I}$ where f "(x) does not exist. If two adjoining intervals change concavity the $\mathbf{x_I}$ between them is an inflection point, *provided* substitution of $\mathbf{x_I}$ into f(x) will yield a y value.

I) *Sketch*

- From the information gathered in **A** through **H**, sketch the curve. If unsure about a particular area try a simple substitution of an x value in f(x) to obtain the y value.