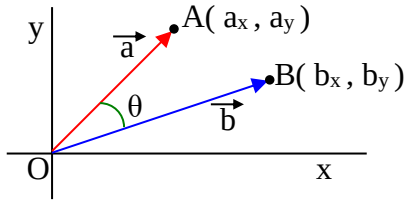
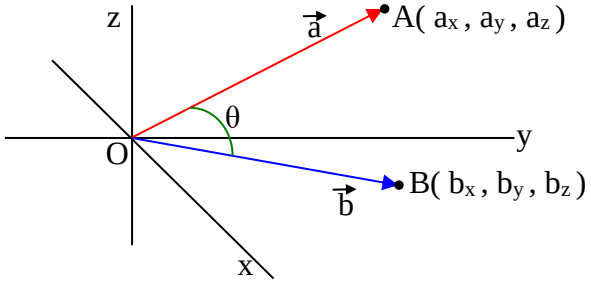
Brebeuf College — Calculus Vector Formulas — Mr. Ryan





1) $\vec{a}$, $\vec{b}$ **Collinear:** $\vec{a} = \mathbf{K}\vec{b}$ $(a_x, a_y) = \mathbf{K}(b_x, b_y)$ $(a_x, a_y, a_z) = \mathbf{K}(b_x, b_y, b_z)$

2) $Comp_{\vec{b}}\vec{a} = |\vec{a}|\cos(\theta)$ $Comp_{\vec{b}}\vec{a} = \dfrac{\vec{a}\bullet\vec{b}}{|\vec{b}|}$ $Proj_{\vec{b}}\vec{a} = |\vec{a}|\cos(\theta)\dfrac{\vec{b}}{|\vec{b}|}$ $Proj_{\vec{b}}\vec{a} = \dfrac{\vec{a}\bullet\vec{b}}{|\vec{b}|^2}\vec{b}$

3) **Dot (Scalar) Product** $\vec{a}\bullet\vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$ $\vec{a}\bullet\vec{b} = a_xb_x + a_yb_y$ $\vec{a}\bullet\vec{b} = a_xb_x + a_yb_y + a_zb_z$

1) $\vec{a}\bullet\vec{a} = |\vec{a}|^2$ **2)** $\vec{a}\bullet\vec{b} = \vec{b}\bullet\vec{a}$ **3)** $\vec{a}\bullet\vec{b} = 0$ **IFF** $\vec{a}\perp\vec{b}$ **4)** $\vec{a}\bullet(\vec{b}+\vec{c}) = \vec{a}\bullet\vec{b} + \vec{a}\bullet\vec{c}$ **5)** $\mathbf{K}\vec{a}\bullet\mathbf{M}\vec{b} = \mathbf{KM}(\vec{a}\bullet\vec{b})$

4) **Cross (Vector) Product** $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin(\theta)$ $\vec{a}\times\vec{b} = (a_yb_z - a_zb_y,\ a_zb_x - a_xb_z,\ a_xb_y - a_yb_x)$

1) $\vec{a}\times\vec{a} = \vec{0}$ **2)** $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$ **3)** $\vec{a}\times\vec{b} = \vec{0}$ **IFF** $\vec{a} = \mathbf{K}\vec{b}$ **4)** $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$ **5)** $\mathbf{K}\vec{a}\times\mathbf{M}\vec{b} = \mathbf{KM}(\vec{a}\times\vec{b})$

note: $\vec{a}\times\vec{b}$ is a new vector that is perpendicular to the plane conaining vectors $\vec{a}$ and $\vec{b}$ and forms a RIGHT hand system.

5) **Angle**: $\cos(\theta) = \dfrac{\vec{a}\bullet\vec{b}}{|\vec{a}||\vec{b}|}$ $\cos(\theta) = \dfrac{a_xb_x + a_yb_y}{\sqrt{a_x^2 + a_y^2}\sqrt{b_x^2 + b_y^2}}$ $\cos(\theta) = \dfrac{a_xb_x + a_yb_y + a_zb_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2}}$

$\sin(\theta) = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ $\sin(\theta) = \dfrac{\sqrt{(a_yb_z - a_zb_y)^2 + (a_zb_x - a_xb_z)^2 + (a_xb_y - a_yb_x)^2}}{\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2}}$

6) **Area of Parallogram** formed by $\vec{a}$, $\vec{b}$: Area = $|\vec{a}\times\vec{b}|$ Area = $\sqrt{(a_yb_z - a_zb_y)^2 + (a_zb_x - a_xb_z)^2 + (a_xb_y - a_yb_x)^2}$

7) **Triple Scalar Product**: $\vec{a}\times\vec{b}\bullet\vec{c} = (a_yb_z - a_zb_y)c_x + (a_zb_x - a_xb_z)c_y + (a_xb_y - a_yb_x)c_z$

8) **Volume of Parallepiped**: Vol = $|\vec{a}\times\vec{b}\bullet\vec{c}|$ Vol = $|(a_yb_z - a_zb_y)c_x + (a_zb_x - a_xb_z)c_y + (a_xb_y - a_yb_x)c_z|$
formed by $\vec{a}$, $\vec{b}$ and $\vec{c}$

NOTE: if volume is **0** then the three vectors are **COPLANAR**

9) **Resolve Vector Componets:**

a) along x axis: $|\vec{a}|\cos(\theta)$
b) along y axis: $|\vec{a}|\sin(\theta)$