Brebeuf College — Calculus Limit - Rules — Mr. Ryan

Rules:

1) $\lim_{x\to a} [\, f(x) \pm g(x) \,] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

2) $\lim_{x\to a} c\, f(x) = c \lim_{x\to a} f(x)$

3) $\lim_{x\to a} [\, f(x) \bullet g(x) \,] = \lim_{x\to a} f(x) \bullet \lim_{x\to a} g(x)$

4) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $\quad \lim_{x\to a} g(x) \neq 0$

5) $\lim_{x\to a} [\, f(x) \,]^n = [\, \lim_{x\to a} f(x) \,]^n$

6) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ $\quad$ f(x) > 0 if n is even

Types:

1) **Limits of Constants:** $\lim_{x\to a} 6y + 2 = 6y + 2$

2) **Direct Substitution**: $\lim_{x\to 2} \frac{x+1}{x+2} = \frac{(2)+1}{(2)+2} = \frac{3}{4}$

3) **Left-Right Limits**: $\lim_{x\to 0^+} \frac{\sin(x)}{x} = 1$

A) Algebraic solution if possible takes Precedence.
B) Use knowledge of the graph(s) to work it out.
C) Use Calculator to check close values.

4) $\frac{1}{0}$ **Type Limits**: $\lim_{x\to 2} \frac{1}{x-2}$

A) Use Left/Right limits to check for infinity
B) Usually result in Vertical Asymptotes.

5) $\frac{0}{0}$ **Type Limits**:

A) Factor and Cancel $\quad \lim_{x\to 2} \frac{x^2-4}{x-2}$

B) Common Denominator $\quad \lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

C) Rationalize $\quad \lim_{x\to 0} \frac{\sqrt{x^2+9} - 3}{x^2}$

6) **Infinity Limits**: $\lim_{x\to\infty} \frac{5x^2-4x}{3x^2-2} = \frac{5}{3}$

A) Divide terms by highest power of x
B) Usually results in Horizontal Asymptotes.

7) **Squeeze Theorem**: $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$

Squeeze between $-x^2 < x^2 \sin(\frac{1}{x}) < x^2$

8) **Delta-Epsilon**: $\lim_{x\to a} f(x) = L$

for every $\varepsilon > 0$ there is a $\delta > 0$ such that:
If $0 < |\, x - a \,| < \delta$ then $|\, f(x) - L \,| < \varepsilon$

9) **L'Hospital's Rule**: For $\frac{0}{0}$ or $\frac{\infty}{\infty}$ forms