To Sketch the Curve $y = f(x)$

A) *Domain*

- Determine the set of values of x for which f(x) is defined.

B) *Intercepts*

- Find the y intercept by setting x = 0 and solving for y (0 , y)
- Find the x intercept by setting y = 0 and solving for x (x , 0) *** Do not attempt if too difficult**.

C) *Symmetry*

- If f(-x) = f(x) then f(x) is even and symmetrical about the y - axis.
- If f(-x) = -f(x) then f(x) is odd and symmetrical about the origin.

D) *Asymptotes*

- **Hole:** If $(x - x_h)$ can be cancelled on the top and bottom of f(x) then $\boxed{x = x_h}$ is a hole.

- **Vertical:** Equate the denominator of f(x) to zero after cancelling common factors to find $\boxed{x = x_v}$. Evaluate $\lim_{x \to x_v^-} f(x)$ and $\lim_{x \to x_v^+} f(x)$ to identify either $-\infty$ or ∞.

- **Horizontal:** If $\lim_{x \to -\infty} f(x) = \mathbf{L}$ or $\lim_{x \to \infty} f(x) = \mathbf{L}$ then $\boxed{y = \mathbf{L}}$ is a horizontal asymptote.

- **Slant:** The line $\boxed{y = \mathbf{mx + b}}$ is a slant asymptote when $\lim_{x \to \pm\infty} [\, f(x) - (\mathbf{mx + b})\,] = 0$

E) *Intervals*

- Find all Type I critical numbers x_c by setting f '(x) = 0 and solving for x.
- Find all Type II critical numbers x_c where f '(x) does not exist.
- Set up a chart of intervals using these critical numbers.
- If f '(x) > 0 the interval is increasing and if f '(x) < 0 the interval is decreasing.

F) *Max/Min*

- *First Derivative Test:* If **$f(x_c)$ exists** and interval changes from inc → dec $[x_c , f(x_c)]$ is a Local Max but if interval changes from dec → inc then $[x_c , f(x_c)]$ is a Local Min.

- *Second Derivative Test:* If **$f(x_c)$ & $f'(x_c)$ exist** and $f''(x_c) > 0$ then $[x_c , f(x_c)]$ is a Local Min but if $f''(x_c) < 0$ then $[x_c , f(x_c)]$ is a Local Max. If $f''(x_c) = 0$ or $f''(x_c)$ doesn't exist, use the *First Derivative Test*.

- *Inflection point:* Type I x_c **no** change inc → dec or dec → inc occurs.
- *Vert Tangent:* Type II x_c , $f(x_c)$ = exists , **no** change inc → dec or dec → inc occurs.
- *Cusp:* Type II x_c , $f(x_c)$ = exists , **a** change inc → dec or dec → inc occurs.

G) *Concavity*

- Find all Type I inflection numbers x_i by setting f "(x) = 0 and solving for x.
- Find all Type II inflection numbers x_i where f "(x) does not exist.
- Set up a chart of intervals using these inflection numbers.
- If f "(x) > 0 the interval is Concave Up and if f "(x) < 0 the interval is Concave Down.

H) *Inflection*

- If **$f(x_i)$ exists** and an interval changes concavity then $[x_i , f(x_i)]$ is an inflection point.

I) *Sketch*

- From the information gathered in **A** through **H**, sketch the curve. If unsure about a particular area try a simple substitution of an x value in f(x) to obtain the y value.