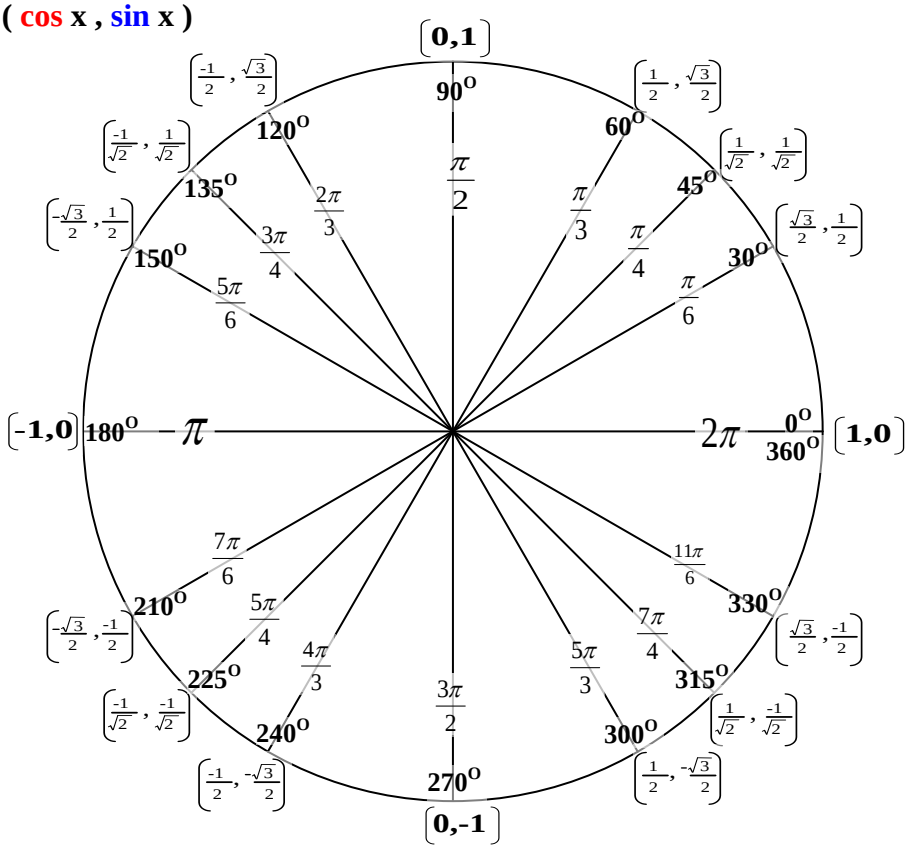
Brebeuf College — Trigonometric Help Sheet — B. Ryan

(cos x , sin x)



$\sin x = \frac{\text{Opp}}{\text{Hyp}}$ $\cos x = \frac{\text{Adj}}{\text{Hyp}}$ $\tan x = \frac{\text{Opp}}{\text{Adj}}$

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

	sin	cos	tan	csc	sec	cot
(-x)	- sin x	cos x	- tan x	- csc x	sec x	- cot x
$\left(\frac{\pi}{2} - x\right)$	cos x	sin x	cot x	sec x	csc x	tan x
$\left(\frac{\pi}{2} + x\right)$	cos x	- sin x	- cot x	sec x	- csc x	- tan x
$(\pi - x)$	sin x	- cos x	- tan x	csc x	- sec x	- cot x
$(\pi + x)$	- sin x	- cos x	tan x	- csc x	- sec x	cot x
$\left(\frac{3\pi}{2} - x\right)$	- cos x	- sin x	cot x	- sec x	- csc x	tan x
$\left(\frac{3\pi}{2} + x\right)$	- cos x	sin x	- cot x	- sec x	csc x	- tan x
$(2\pi - x)$	- sin x	cos x	- tan x	- csc x	sec x	- cot x

Conversion: ($\pi / 180^{O}$) X Degrees = Radians

($180^{O} / \pi$) X Radians = Degrees

Pythagorean: $\sin^2 x + \cos^2 x = 1$

$\sec^2 x - \tan^2 x = 1$

$\csc^2 x - \cot^2 x = 1$

Double Angle: $\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Addition Formulas:

$\sin (x + y) = \sin x \cos y + \cos x \sin y$

$\sin (x - y) = \sin x \cos y - \cos x \sin y$

$\cos (x + y) = \cos x \cos y - \sin x \sin y$

$\cos (x - y) = \cos x \cos y + \sin x \sin y$

$\tan (x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan (x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Substitutions: $\sin x = \sqrt{\frac{1 - \cos 2x}{2}}$ $\cos x = \sqrt{\frac{1 + \cos 2x}{2}}$

Various Formulas:

$\sin x \sin y = \frac{\cos (x - y) - \cos (x + y)}{2}$

$\cos x \cos y = \frac{\cos (x + y) + \cos (x - y)}{2}$

$\sin x \cos y = \frac{\sin (x + y) + \sin (x - y)}{2}$

$\sin x + \sin y = 2 \sin \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$

$\sin x - \sin y = 2 \cos \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$

$\cos x + \cos y = 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$

$\cos x - \cos y = -2 \sin \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$

$\cos (x + y) \cos (x - y) = \cos^2 x - \sin^2 y$

$\sin (x + y) \sin (x - y) = \cos^2 y - \cos^2 x$

$\tan (x + y) \tan (x - y) = \frac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$

Limits: $\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

Derivatives:

$\frac{d}{dx} \sin x = \cos x$ $\frac{d}{dx} \cos x = -\sin x$ $\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \csc x = -\csc x \cot x$ $\frac{d}{dx} \sec x = \sec x \tan x$ $\frac{d}{dx} \cot x = -\csc^2 x$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$